

Class 12 Maths: Complete Formula Sheet

All 13 NCERT chapters (Part I and Part II) · CBSE syllabus 2026-27 · NCERT textbook (Reprint 2026-27)

How to use this sheet. Plain boxes cover the CBSE 2026-27 board-exam syllabus. Grey boxes marked $\diamond$ **Not in 2026-27 syllabus** are standard results that the current syllabus does not list. They are kept for reference, older papers and JEE. C is the constant of integration, and $\log$ means the natural logarithm $\log_e$.

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1 Relations and Functions

TYPES OF RELATIONS (R IN A SET A)

- Empty: $R = \varphi$; universal: $R = A \times A$
- Reflexive: $(a, a) \in R$ for every $a \in A$
- Symmetric: $(a, b) \in R \Rightarrow (b, a) \in R$
- Transitive: $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
- Equivalence: reflexive, symmetric and transitive
- Equivalence class $[a]$ = set of all elements related to a ; the classes are disjoint and their union is A

TYPES OF FUNCTIONS ($f : X \rightarrow Y$)

- One-one (injective): $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
- Onto (surjective): every $y \in Y$ has some $x \in X$ with $f(x) = y$, i.e. range = codomain
- Bijective: both one-one and onto
- For a finite set X , $f : X \rightarrow X$ is one-one $\iff$ it is onto (not true for infinite sets)

$\diamond$ NOT IN 2026-27 SYLLABUS: COMPOSITION AND INVERSE

- $f : A \rightarrow B$, $g : B \rightarrow C$: $(g \circ f)(x) = g(f(x))$
- f is invertible $\iff f$ is one-one and onto; then $g = f^{-1}$ satisfies $g \circ f = I_X$, $f \circ g = I_Y$

2 Inverse Trigonometric Functions

DOMAINS AND PRINCIPAL VALUE BRANCHES

Function	Domain	Range (principal branch)
$\sin^{-1} x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\csc^{-1} x$	$\mathbb{R} - (-1, 1)$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
$\sec^{-1} x$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$\tan^{-1} x$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\cot^{-1} x$	$\mathbb{R}$	$(0, \pi)$

KEY FACTS

- $\sin^{-1} x \neq (\sin x)^{-1}$; in fact $(\sin x)^{-1} = \frac{1}{\sin x}$
- $y = \sin^{-1} x \iff x = \sin y$ (y in the principal branch)
- Principal value = the value lying in the principal branch
- $\sin(\sin^{-1} x) = x$ for $x \in [-1, 1]$
- $\sin^{-1}(\sin x) = x$ for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- $\cos^{-1}(\cos x) = x$ for $x \in [0, \pi]$; $\tan^{-1}(\tan x) = x$ for $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$
- The graph of $y = \sin^{-1} x$ is the mirror image of the principal branch of $y = \sin x$ in the line $y = x$ (same for the others)



◇ NOT IN 2026-27 SYLLABUS: STANDARD IDENTITIES

- $\sin^{-1}(-x) = -\sin^{-1} x$, $\tan^{-1}(-x) = -\tan^{-1} x$, $\csc^{-1}(-x) = -\csc^{-1} x$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $\cot^{-1}(-x) = \pi - \cot^{-1} x$, $\sec^{-1}(-x) = \pi - \sec^{-1} x$
- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ ($|x| \leq 1$); $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$; $\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$ ($|x| \geq 1$)
- $\sin^{-1} \frac{1}{x} = \csc^{-1} x$ ($|x| \geq 1$); $\cos^{-1} \frac{1}{x} = \sec^{-1} x$ ($|x| \geq 1$); $\tan^{-1} \frac{1}{x} = \cot^{-1} x$ ($x > 0$)
- $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$ for $|x| < 1$

3 Matrices

TYPES OF MATRICES

- Order $m \times n$: m rows, n columns; $A = [a_{ij}]_{m \times n}$
- Column $[a_{ij}]_{m \times 1}$; row $[a_{ij}]_{1 \times n}$; square: $m = n$
- Diagonal: $a_{ij} = 0$ for $i \neq j$
- Scalar: diagonal with all $a_{ii} = k$
- Identity I : $a_{ii} = 1$, $a_{ij} = 0$ for $i \neq j$
- Zero matrix O : all entries 0
- $A = B$ iff same order and $a_{ij} = b_{ij}$ for all i, j

ADDITION AND SCALAR MULTIPLICATION

- $A + B = [a_{ij} + b_{ij}]$ (same order only); $kA = [k a_{ij}]$
- $-A = (-1)A$, $A - B = A + (-1)B$
- $A + B = B + A$; $(A + B) + C = A + (B + C)$
- $A + O = A$; $A + (-A) = O$
- $k(A + B) = kA + kB$; $(k + l)A = kA + lA$

MULTIPLICATION

- AB is defined only if columns of $A =$ rows of B :
 $A_{m \times n} B_{n \times p} = C_{m \times p}$
- $c_{ik} = \sum_{j=1}^n a_{ij} b_{jk}$ (row i of $A \cdot$ column k of B)
- $A(BC) = (AB)C$; $A(B + C) = AB + AC$; $(A + B)C = AC + BC$
- $AI = IA = A$
- In general $AB \neq BA$; and $AB = O$ is possible with $A \neq O, B \neq O$

TRANSPOSE, SYMMETRIC, SKEW SYMMETRIC

- $A' = A^T = [a_{ji}]_{n \times m}$
- $(A')' = A$; $(kA)' = kA'$; $(A + B)' = A' + B'$;
 $(AB)' = B'A'$
- Symmetric: $A' = A$; skew symmetric: $A' = -A$ (diagonal entries are 0)
- $A + A'$ is symmetric, $A - A'$ is skew symmetric
- Every square matrix: $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

INVERTIBLE MATRICES

- If $AB = BA = I$, then $B = A^{-1}$ (and $A = B^{-1}$); the inverse, if it exists, is unique
- $(AB)^{-1} = B^{-1}A^{-1}$; $(A^{-1})^{-1} = A$
- 2×2 shortcut: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, provided $ad - bc \neq 0$



4 Determinants

EVALUATING DETERMINANTS

- $|a_{11}| = a_{11}$; $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$
- Expanding along R_1 : $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$
 $\begin{matrix} & + & - & + \\ & - & + & - \\ & + & - & + \end{matrix}$
- Any row or column can be used; sign pattern

MINORS AND COFACTORS

- Minor M_{ij} : determinant left after deleting row i and column j
- Cofactor: $A_{ij} = (-1)^{i+j} M_{ij}$
- $|A|$ = sum of (elements of any row or column) $\times$ (their cofactors), e.g. $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$
- Elements of one row $\times$ cofactors of **another** row sum to 0, e.g. $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$

AREA OF A TRIANGLE

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

- Area is positive, so take the absolute value
- Points are collinear $\iff \Delta = 0$

ADJOINT AND INVERSE

- $\text{adj } A$ = transpose of the matrix of cofactors:
 $\text{adj } A = [A_{ij}]'$
- $A(\text{adj } A) = (\text{adj } A)A = |A| I$
- Singular if $|A| = 0$; non-singular if $|A| \neq 0$
- A is invertible $\iff |A| \neq 0$; $A^{-1} = \frac{1}{|A|} \text{adj } A$

USEFUL RESULTS (A OF ORDER n)

- $|kA| = k^n |A|$
- $|\text{adj } A| = |A|^{n-1}$
- $|AB| = |A||B|$; $|A'| = |A|$; $|A^{-1}| = \frac{1}{|A|}$

SYSTEM OF LINEAR EQUATIONS $AX = B$

Condition	Result
$ A \neq 0$	Consistent, unique solution $X = A^{-1}B$
$ A = 0$ and $(\text{adj } A)B \neq O$	Inconsistent, no solution
$ A = 0$ and $(\text{adj } A)B = O$	May or may not be consistent (if consistent, infinitely many solutions)

For $a_1x + b_1y + c_1z = d_1$ etc.: $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$. Consistent = at least one solution exists.



5 Continuity and Differentiability

CONTINUITY

- f is continuous at c if $\lim_{x \rightarrow c} f(x) = f(c)$, i.e. LHL = RHL = $f(c)$
- Sum, difference, product and quotient (where $g(x) \neq 0$) of continuous functions are continuous
- Composition of continuous functions is continuous
- Polynomials, $\sin x$, $\cos x$, $|x|$, e^x are continuous everywhere; $\log x$ for $x > 0$

DIFFERENTIABILITY

- f is differentiable at c if LHD = RHD:

$$\lim_{h \rightarrow 0^-} \frac{f(c+h) - f(c)}{h} = \lim_{h \rightarrow 0^+} \frac{f(c+h) - f(c)}{h}$$
- Differentiable $\Rightarrow$ continuous; the converse is false (e.g. $|x|$ at $x = 0$)

STANDARD DERIVATIVES

$f(x)$	x^n	$\sin x$	$\cos x$	$\tan x$	$\cot x$	$\sec x$	$\csc x$
$f'(x)$	nx^{n-1}	$\cos x$	$-\sin x$	$\sec^2 x$	$-\csc^2 x$	$\sec x \tan x$	$-\csc x \cot x$
$f(x)$	e^x	a^x	$\log x$	$\log_a x$	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
$f'(x)$	e^x	$a^x \log a$	$\frac{1}{x}$	$\frac{1}{x \log a}$	$\frac{1}{\sqrt{1-x^2}}$	$-\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$

Valid in the appropriate domains: $x > 0$ for logs, $|x| < 1$ for $\sin^{-1} x$ and $\cos^{-1} x$.

RULES

- $(u \pm v)' = u' \pm v'$; $(uv)' = u'v + uv'$
- $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, $v \neq 0$
- Chain rule: $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$
- $\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$
- Implicit: differentiate both sides w.r.t. x , treating y as a function of x (e.g. $\frac{d}{dx} y^2 = 2y \frac{dy}{dx}$)
- Parametric $x = f(t)$, $y = g(t)$: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, $\frac{dx}{dt} \neq 0$
- Second order: $\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$, also written y'' or y_2

LOGARITHMS AND LOG DIFFERENTIATION

- $\log_b a = x \iff b^x = a$; $\log_a p = \frac{\log_b p}{\log_b a}$
- $\log(xy) = \log x + \log y$; $\log \frac{x}{y} = \log x - \log y$;
 $\log x^n = n \log x$
- $e^{\log x} = x$ ($x > 0$); $a^x = e^{x \log a}$
- For $y = [u(x)]^{v(x)}$: take $\log y = v \log u$, then $\frac{1}{y} \frac{dy}{dx} = v' \log u + v \frac{u'}{u}$ (needs $u > 0$)

◇ NOT IN 2026-27 SYLLABUS: OTHER INVERSE TRIG DERIVATIVES

- $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$; $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$; $\frac{d}{dx} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$ ($|x| > 1$)



6 Application of Derivatives

RATE OF CHANGE

- $\frac{dy}{dx}$ = rate of change of y w.r.t. x ; $\frac{dy}{dx}\bigg|_{x=x_0}$ = rate at $x = x_0$
- If $x = f(t)$, $y = g(t)$: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, $\frac{dx}{dt} \neq 0$
- Marginal cost = $\frac{dC}{dx}$, marginal revenue = $\frac{dR}{dx}$

INCREASING AND DECREASING

- Increasing on I : $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$; strictly: $f(x_1) < f(x_2)$
- $f'(x) > 0$ on $(a, b) \Rightarrow$ strictly increasing on $[a, b]$
- $f'(x) < 0$ on $(a, b) \Rightarrow$ strictly decreasing on $[a, b]$
- $f'(x) = 0$ on $(a, b) \Rightarrow$ constant on $[a, b]$
- (f continuous on $[a, b]$, differentiable on (a, b))

CRITICAL POINTS AND FIRST DERIVATIVE TEST

- Critical point c : $f'(c) = 0$ or f not differentiable at c
- f' changes $+$ $\rightarrow$ $-$ through c : local maximum
- f' changes $-$ $\rightarrow$ $+$ through c : local minimum
- No sign change: neither (point of inflexion)

SECOND DERIVATIVE TEST ($f'(c) = 0$)

- $f''(c) < 0$: local maximum, value $f(c)$
- $f''(c) > 0$: local minimum, value $f(c)$
- $f''(c) = 0$: test fails, so use the first derivative test

ABSOLUTE MAXIMUM AND MINIMUM ON A CLOSED INTERVAL $[a, b]$

- Step 1: find all critical points in the interval. Step 2: take the end points a and b .
- Step 3: evaluate f at all these points. Step 4: the largest value is the absolute maximum, the smallest is the absolute minimum.

7 Integrals

BASICS

- If $\frac{d}{dx}F(x) = f(x)$, then $\int f(x) dx = F(x) + C$
- $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$
- $\int k f(x) dx = k \int f(x) dx$
- $\frac{d}{dx} \int f(x) dx = f(x)$

INTEGRALS VIA TRIG IDENTITIES

- $\sin^2 x = \frac{1 - \cos 2x}{2}$, $\cos^2 x = \frac{1 + \cos 2x}{2}$
- $\sin^3 x = \frac{3 \sin x - \sin 3x}{4}$, $\cos^3 x = \frac{3 \cos x + \cos 3x}{4}$
- $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$, and the other product-to-sum formulas

STANDARD INTEGRALS

$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$	$\int \frac{1}{x} dx = \log x + C$	$\int e^x dx = e^x + C$
$\int a^x dx = \frac{a^x}{\log a} + C$	$\int \sin x dx = -\cos x + C$	$\int \cos x dx = \sin x + C$
$\int \sec^2 x dx = \tan x + C$	$\int \csc^2 x dx = -\cot x + C$	$\int \sec x \tan x dx = \sec x + C$
$\int \csc x \cot x dx = -\csc x + C$	$\int \tan x dx = \log \sec x + C$	$\int \cot x dx = \log \sin x + C$
$\int \sec x dx = \log \sec x + \tan x + C$	$\int \csc x dx = \log \csc x - \cot x + C$	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C = -\cos^{-1} x + C$
$\int \frac{dx}{1+x^2} = \tan^{-1} x + C = -\cot^{-1} x + C$		



INTEGRALS OF PARTICULAR FUNCTIONS

$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left \frac{x-a}{x+a} \right + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$
$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left x + \sqrt{x^2 - a^2} \right + C$	$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left x + \sqrt{x^2 + a^2} \right + C$
$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$	
$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left x + \sqrt{x^2 + a^2} \right + C$	
$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left x + \sqrt{x^2 - a^2} \right + C$	

QUADRATIC DENOMINATORS

- For $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$,
 $\int \sqrt{ax^2 + bx + c} dx$, complete the square:
 $ax^2 + bx + c = a \left[\left(x + \frac{b}{2a} \right)^2 + \left(\frac{c}{a} - \frac{b^2}{4a^2} \right) \right]$
- For $\int \frac{px + q}{ax^2 + bx + c} dx$ or $\int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx$:
write $px + q = A(2ax + b) + B$ and find A, B by
comparing coefficients

INTEGRATION BY PARTS

- $$\int f_1 f_2 dx = f_1 \int f_2 dx - \int \left[\frac{df_1}{dx} \int f_2 dx \right] dx$$
- Choose f_2 as the function whose integral you know.
Common order for f_1 (ILATE): Inverse trig, Log,
Algebraic, Trig, Exponential
 - $\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$

PARTIAL FRACTIONS (DEGREE OF NUMERATOR < DEGREE OF DENOMINATOR; OTHERWISE DIVIDE FIRST)

Rational function	Partial fractions
$\frac{px + q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{px + q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{px^2 + qx + r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
$\frac{px^2 + qx + r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
$\frac{px^2 + qx + r}{(x-a)(x^2 + bx + c)}$ ($x^2 + bx + c$ cannot be factorised)	$\frac{A}{x-a} + \frac{Bx + C}{x^2 + bx + c}$



DEFINITE INTEGRALS

- $\int_a^b f(x) dx$ = area bounded by $y = f(x)$, the x-axis and $x = a$, $x = b$
- First fundamental theorem: if $A(x) = \int_a^x f(t) dt$, then $A'(x) = f(x)$
- Second fundamental theorem: $\int_a^b f(x) dx = \left[F(x) \right]_a^b = F(b) - F(a)$, where $F' = f$
- With substitution, change the limits too

PROPERTIES OF DEFINITE INTEGRALS

- P_0 : $\int_a^b f(x) dx = \int_a^b f(t) dt$
- P_1 : $\int_a^b f(x) dx = -\int_b^a f(x) dx$; $\int_a^a f(x) dx = 0$
- P_2 : $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$
- P_3 : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$
- P_4 : $\int_0^a f(x) dx = \int_0^a f(a-x) dx$
- P_5 : $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$
- P_6 : $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$ if $f(2a-x) = f(x)$; $= 0$ if $f(2a-x) = -f(x)$
- P_7 : $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ if f is even; $= 0$ if f is odd

8 Application of Integrals**AREA UNDER A CURVE**

- Bounded by $y = f(x)$, the x-axis, $x = a$, $x = b$ ($b > a$): Area = $\int_a^b y dx = \int_a^b f(x) dx$
- Bounded by $x = \varphi(y)$, the y-axis, $y = c$, $y = d$:
Area = $\int_c^d x dy = \int_c^d \varphi(y) dy$
- If the curve is below the x-axis, the integral is negative: take $\left| \int_a^b f(x) dx \right|$
- If the curve crosses the axis, split the interval and add the absolute values

USEFUL RESULTS (STANDARD CURVES)

- Circle $x^2 + y^2 = a^2$: area = πa^2
- Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$: area = πab
- Parabola $y^2 = 4ax$ and its latus rectum $x = a$: area = $\frac{8a^2}{3}$
- Use symmetry: find the area in one quadrant and multiply

9 Differential Equations**BASICS**

- Order = order of the highest derivative present
- Degree (defined only if the equation is a polynomial in its derivatives) = highest power of the highest-order derivative
- General solution: as many arbitrary constants as the order
- Particular solution: no arbitrary constants (found using given conditions)

VARIABLES SEPARABLE

- $\frac{dy}{dx} = g(x)h(y)$: write $\frac{dy}{h(y)} = g(x) dx$, then
 $\int \frac{dy}{h(y)} = \int g(x) dx + C$



HOMOGENEOUS EQUATIONS

- $F(x, y)$ is homogeneous of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$
- $\frac{dy}{dx} = F(x, y)$ is homogeneous if F has degree 0
- Put $y = vx$: $\frac{dy}{dx} = v + x \frac{dv}{dx}$, then separate variables
- For $\frac{dx}{dy} = G(x, y)$, put $x = vy$: $\frac{dx}{dy} = v + y \frac{dv}{dy}$

LINEAR EQUATIONS

- $\frac{dy}{dx} + Py = Q$ (P, Q functions of x or constants):
I.F. = $e^{\int P dx}$, $y \cdot \text{I.F.} = \int (Q \times \text{I.F.}) dx + C$
 - $\frac{dx}{dy} + P_1x = Q_1$ (P_1, Q_1 functions of y or constants):
I.F. = $e^{\int P_1 dy}$, $x \cdot \text{I.F.} = \int (Q_1 \times \text{I.F.}) dy + C$
- Useful: $e^{\log f(x)} = f(x)$.

10 Vector Algebra**BASICS**

- Position vector of $P(x, y, z)$: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$,
 $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$
- $\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
- Unit vector along $\vec{a}$: $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$
- Direction cosines of $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$: $l = \frac{a}{r}$, $m = \frac{b}{r}$, $n = \frac{c}{r}$; a, b, c are direction ratios
- $l^2 + m^2 + n^2 = 1$; $\vec{r} = r(l\hat{i} + m\hat{j} + n\hat{k})$

TYPES AND OPERATIONS

- Zero, unit, coinitial, collinear (parallel to the same line), equal (same magnitude and direction), negative vectors
- Triangle law: $\vec{AB} + \vec{BC} = \vec{AC}$; sides of a triangle taken in order add to $\vec{0}$
- Parallelogram law: the sum of two coinitial vectors is the diagonal
- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$; $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
- $\lambda \vec{a}$ has magnitude $|\lambda||\vec{a}|$; same direction if $\lambda > 0$, opposite if $\lambda < 0$
- $\vec{a}, \vec{b}$ collinear $\iff \vec{b} = \lambda \vec{a} \iff \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3}$

COMPONENTS

- For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$:
- $\vec{a} \pm \vec{b} = (a_1 \pm b_1)\hat{i} + (a_2 \pm b_2)\hat{j} + (a_3 \pm b_3)\hat{k}$
- $\lambda \vec{a} = \lambda a_1\hat{i} + \lambda a_2\hat{j} + \lambda a_3\hat{k}$
- $\vec{a} = \vec{b} \iff a_1 = b_1, a_2 = b_2, a_3 = b_3$

SECTION FORMULA (POSITION VECTORS $\vec{a}, \vec{b}$)

- Internal, ratio $m : n$: $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n}$
- External, ratio $m : n$: $\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n}$
- Mid-point: $\vec{r} = \frac{\vec{a} + \vec{b}}{2}$

SCALAR (DOT) PRODUCT

- $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta = a_1b_1 + a_2b_2 + a_3b_3$
- $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$
- $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$ (non-zero vectors)
- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$; $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$
- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$; $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$; $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- Projection of $\vec{a}$ on $\vec{b}$: $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \vec{a} \cdot \hat{b}$

VECTOR (CROSS) PRODUCT

- $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \theta \hat{n}$, $\hat{n} \perp$ both, right-handed
- $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
- $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$; $\vec{a} \parallel \vec{b} \iff \vec{a} \times \vec{b} = \vec{0}$
- $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$; $\hat{i} \times \hat{i} = \vec{0}$
- $\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$
- Unit vector perpendicular to both: $\pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$



AREAS

- Parallelogram with adjacent sides $\vec{a}, \vec{b}$: $|\vec{a} \times \vec{b}|$; triangle with adjacent sides $\vec{a}, \vec{b}$: $\frac{1}{2}|\vec{a} \times \vec{b}|$
- Parallelogram with diagonals $\vec{d}_1, \vec{d}_2$: $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

11 Three Dimensional Geometry**DIRECTION COSINES AND RATIOS**

- DCs l, m, n = cosines of the angles with the positive x, y, z axes; $l^2 + m^2 + n^2 = 1$
- DRs a, b, c are proportional to the DCs:

$$l = \frac{\pm a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{\pm b}{\sqrt{a^2 + b^2 + c^2}}, n = \frac{\pm c}{\sqrt{a^2 + b^2 + c^2}}$$

- Line through $P(x_1, y_1, z_1), Q(x_2, y_2, z_2)$: DRs $x_2 - x_1, y_2 - y_1, z_2 - z_1$; DCs are these divided by PQ

EQUATION OF A LINE

- Through $\vec{a}$, parallel to $\vec{b}$: $\vec{r} = \vec{a} + \lambda \vec{b}$
- Cartesian (through (x_1, y_1, z_1) , DRs a, b, c):

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

(with DCs: replace a, b, c by l, m, n)

- Through two points $\vec{a}, \vec{b}$: $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$
Cartesian: $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

ANGLE BETWEEN TWO LINES (θ ACUTE)

- Vector form $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1, \vec{r} = \vec{a}_2 + \mu \vec{b}_2$: $\cos \theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$
- With DCs: $\cos \theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$; with DRs: $\cos \theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$
- Perpendicular: $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$; parallel: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

SHORTEST DISTANCE

- Skew lines: neither parallel nor intersecting (they lie in different planes). The shortest distance is along the common perpendicular.

- Skew lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1, \vec{r} = \vec{a}_2 + \mu \vec{b}_2$: $d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

- Cartesian form:

$$d = \frac{\left| \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \right|}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}}$$

- Parallel lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}, \vec{r} = \vec{a}_2 + \mu \vec{b}$: $d = \left| \frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|} \right|$
- The lines intersect $\iff d = 0$ (and they are not parallel)



12 Linear Programming

TERMINOLOGY

- Objective function: $Z = ax + by$ (to be maximised or minimised)
- Constraints: linear inequalities; non-negativity: $x \geq 0, y \geq 0$
- Feasible region: the common region of all constraints; each point in it is a feasible solution
- Optimal solution: the feasible point giving the best value of Z
- Infeasible: no point satisfies all constraints

CORNER POINT THEOREMS

- If an optimal value exists, it occurs at a corner point (vertex) of the feasible region
- If the feasible region is bounded, Z has both a maximum and a minimum, each at a corner point
- An unbounded region may have no maximum or minimum

CORNER POINT METHOD

1. Draw the feasible region and find its corner points. 2. Evaluate Z at each corner point. Let M be the largest value and m the smallest.
 3. Bounded region: M is the maximum and m the minimum.
 4. Unbounded region: M is the maximum only if the open half-plane $ax + by > M$ has no point in common with the feasible region (otherwise there is no maximum). Similarly, m is the minimum only if $ax + by < m$ has no common point.
- If two adjacent corners give the same optimal value, every point on the segment joining them is also optimal.

13 Probability

CONDITIONAL PROBABILITY

- $P(E | F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$
- Equally likely outcomes: $P(E | F) = \frac{n(E \cap F)}{n(F)}$
- $P(S | F) = P(F | F) = 1; 0 \leq P(E | F) \leq 1$
- $P(E' | F) = 1 - P(E | F)$
- $P((A \cup B) | F) = P(A | F) + P(B | F) - P((A \cap B) | F)$

INDEPENDENT EVENTS

- E, F independent $\iff P(E \cap F) = P(E)P(F)$
- Then $P(E | F) = P(E)$ and $P(F | E) = P(F)$
- If E, F are independent, so are E and F' , E' and F , E' and F'
- $P(\text{at least one of } E, F) = 1 - P(E')P(F')$
- Three events: mutually independent if every pair is independent and $P(E \cap F \cap G) = P(E)P(F)P(G)$
- Independent is not the same as mutually exclusive.

MULTIPLICATION THEOREM

- $P(E \cap F) = P(E)P(F | E), P(E) \neq 0$
- $P(E \cap F) = P(F)P(E | F), P(F) \neq 0$
- $P(E \cap F \cap G) = P(E)P(F | E)P(G | E \cap F)$

TOTAL PROBABILITY AND BAYES' THEOREM

- Partition: $E_1, \dots, E_n$ pairwise disjoint, $E_1 \cup \dots \cup E_n = S$, each $P(E_i) > 0$
- Total probability:

$$P(A) = \sum_{j=1}^n P(E_j)P(A | E_j)$$

- Bayes' theorem:

$$P(E_i | A) = \frac{P(E_i)P(A | E_i)}{\sum_{j=1}^n P(E_j)P(A | E_j)}$$

- $P(E_i)$ are priors, $P(E_i | A)$ are posteriors.

